Input-Output Pairing for Multivariable Systems

Annasaheb K Bandsode [1], Dr. V S Sathe
Department of Chemical Engineering[1] Department of Chemical Engineering [2],
Finolex Academy of Management and Technology, Ratnagiri[1], Dr Babasaheb Ambedkar Technological Univeristy Lonere [2],

Email: akbandsode@gmail.com[1] , vssathe@dbatu.ac.in[2]

Abstract- Many complex industrial processes are multivariable with multiple inputs and multiple outputs. Generally multivariable systems are characterized by complicated cross couplings where control loops sometimes interact and even fight against each other. This poses significant challenges in designing control systems for these processes. Thus interaction analysis is important for multivariable systems to eliminate undesirable interactions among the control loops. Interactions among control loops in a multivariable system have been the subject of much research over the past years. The purpose of this article is to review these methods. Various arrays, indices and methods for selecting best input-output pairings are discussed in this paper.

Index Terms- Multivariable system;, interaction analysis pairing; and relative gain.

1. INTRODUCTION

A multivariable system has multiple inputs and multiple outputs. In case of these systems the task is to control the multiple output variables by using multiple input variables. In designing controllers for multivariable systems, a typical starting point is the use of multiple, independent single-loop controllers, with each controller using one input variable to control a preassigned output variable. But because of the interactions among the process variables, multivariable systems cannot, in general, be treated like multiple, independent, single loop systems. Thus a multiple single-loop control strategy for multivariable system must take interactions into considerations. The design task would be started by considering the possibility of pairing the input and output variables. For a typical $n \times n$ plant there are $n!$ possible input-output pairings. Therefore selection of a good input-output pair is very important task in the design procedure of design of decentralized control system. A correct input-output pair would result in minimum interactions among control loops

Several authors have proposed different methods to measure the interactions of multivariable processes. Bristol introduced the relative gain array (RGA) [1] as a criterion for choosing the best variable pairing, and this measure continues to be one most often used. Niederlinski proposed Niederlinski index [2] which considered the sign of the determinant of the plant as the screening tool. The use of RGA was discussed in details and the theoretical justification for Bristol's rule of avoiding pairings corresponding to negative relative gains was provided [3]. There are many such methods proposed in literature to eliminate undesirable pairing. The purpose of this paper is to discuss some of these methods. Therefore some of the arrays, indices and methods are briefly discussed to understand the importance of input-output pairings.

2. INPUT-OUTPUT PAIRING BASED ON ARRAYS

2.1. *Relative Gain Array*

The relative gain array [RGA] introduced by Bristol in 1966 [1] is widely used in control system design and analysis. Among the advantages of RGA are the following. It requires minimal process information and due to its ratio nature even approximate process models can give useful results. It is independent of control system tuning and process disturbances. Hence this method is cost effective and popular. RGA is a matrix of numbers. The ij^{th} element of the array is the ratio of steady state gain between the i^{th} controlled variable and j^{th} manipulated variable when all other manipulated variables are constant, divided by the steady state gain between the same two variables when all other controlled variables are constant.

$$\lambda_{ij} = \frac{\left[\frac{x_i}{m_j}\right]_{mk}}{\left[\frac{x_i}{m_j}\right]_{xk}} \qquad (1)$$

For a simple multivariable system with equal number of controlled and manipulated variables whose transfer function matrix is $G(s)$, the matrix of steady state process gains is given as-

$$A = G(0) \qquad (2)$$

For this system the RGA is defined as

$$\text{-}\Lambda = A \otimes (A^T)^{-1} \qquad (3)$$

Where $\otimes$ indicates element by element product.

RGA has some drawbacks and in some cases it leads to incorrect conclusions about how the control loops should be paired and how much loop interactions exists. RGA does not consider disturbances and therefore it does not give any insight into these cases. The most important drawback of this array is that it does not consider dynamics and as a result it can lead to incorrect pairings.

In literature many researchers tried to extend the basic RGA definition, with several modifications. A frequency dependent matrix was introduced as [4]

$$\Lambda(s) = P(s) \otimes \left(P(s)^T\right)^{-1} = P(s) \otimes P(s)^{-T} \quad (4)$$

The frequency dependent RGA is very sensitive to modeling errors as an ideal model of the process is usually unknown. Also a classical frequency-based analysis will require to consider and analyze $n(n+1)$ Bode plots, which is very time consuming [5]. Another approach combines the frequency dependent approach with the singular value decomposition of the transfer function matrix representing the process [6].

2.3. Dynamic Relative Gain Array

RGA does not consider dynamics and as a result it can lead to incorrect pairings. Hence a new approach of defining dynamic relative rain array [DRGA] that overcomes this limitation was introduced. The first approach in this area used a transfer function model in place of steady state model used for RGA calculations [7]. In this case the denominator of the DRGA involved achieving perfect control at all frequencies, while the numerator was simply the open loop transfer function. Many of the studies in this field require detailed feedback controller design [8]. A better approach to defining a useful DRGA should involve a relatively little user interaction in the controller design aspect of the analysis. One such approach was presented [9] which assumes the availability of the dynamic process model which used to design a proportional output optimal controller. The DRGA in this case is defined based on the resulting controller gain matrix. In this study the following linear state space dynamic model for the plant was assumed to be available

$$\frac{dx}{dt} = Ax + Bu \quad (5)$$

$$y = Cx \quad (6)$$

The above equations are scaled an written in terms of scaled variables

$$\frac{dx}{dt} = Ax + B_S\bar{u} \quad (7)$$

$$\bar{y} = C_S x \quad (8)$$

[

$\bar{u}$ and $\bar{y}$ are the scaled variables and B_S and C_S are calculated from B and C by suing scale factors.

The proportional optimal controller is obtained by minimizing the following objective function

$$J = \frac{1}{2}\int_0^\infty (\bar{y}^T Q\bar{y} + \bar{u}^T R\bar{u})dt \quad (9)$$

Here Q and R are taken as identity matrices. An output feedback matrix gives the admissible controls

$$\vec{u} = -K\vec{y} \quad (10)$$

Since the controller K is calculated based on the dynamic model of the process hence it contains the information of the process dynamics. K has been assumed to be square matrix. Based on this controller the ij^{th} element of the DRGA is given as-

$$\lambda_{ij} = \frac{\left[\frac{\partial u_i}{\partial y_j}\right]_{uk\neq o,k\neq i}}{\left[\frac{\partial u_i}{\partial y_j}\right]_{uk=0,k\neq i}} \quad (11)$$

Both terms in Eq. [11] give gain of u_i to y_j during a transient in which the process is controlled using the optimal output proportional gain matrix, K. The numerator gives the change in the manipulated variable u_i to change in the measurement, y_j, for the case where the optimal controller is bringing the system back to the origin starting from a random initial state on the unit sphere. The denominator is calculated using the same optimal controller gain matrix, K, used in the numerator

2.2. Relative Omega Array [ROmA]

A tool for selecting right pairing between inputs and outputs based on characteristic frequencies in closed loop and open under perfect control was introduced which was structurally similar to the classical RGA [5]. In classical RGA the variable under test is steady state gain where as in this method it the critical frequency.

$\hat{W}_{ij}$ is defined as a transfer function in case of perfect control and $\hat{\omega}_{ij}$ is the corresponding critical frequency. Then the ratio $\frac{\omega_{ij}}{\hat{\omega}_{ij}}$ is used to create the following new matrix which is a new dominance index.-

$$F = \left\{\frac{\omega_{ij}}{\hat{\omega}_{ij}}\right\} \quad [12]$$

Dominance is guaranteed if a ratio $\frac{\omega_{ij}}{\hat{\omega}_{ij}}$ tends to one.

The matrix RoMA is then given as-

$$\varphi = F \otimes F^{-T} \quad (13)$$

RoMA retains all the properties of classical RGA

2.4. Normalized RGA [NRGA]

NRGA was introduced through the combination of RGA matrix and its selection rules [10]. Using NRGA pairing is interpreted as an assignment problem which is solved by Hungarian algorithm. Hence in this case pair is performed automatically without human intervention which is the case with classical RGA. With this method it is possible to pair adaptively the inputs and outputs in a nonlinear and/or time variable process, where the optimal pairing may change from time to time.

In RGA selection of one of the two pairs with relative gain values in either side of 1 is ambiguous. This ambiguity arises because in both cases one try to select closer values of 1 but definition of closer in subspace [1,0] is different from its definition in subspace$[1, +\infty]$. This problem was dealt with by interpreting "close to 1" by the function

$$f(\lambda) = \begin{cases} 0 & \lambda \leq 0 \\ f_1(\lambda) & 0 < \lambda \leq 1 \\ f_u(\lambda) & 1 < \lambda \end{cases} \quad (14)$$

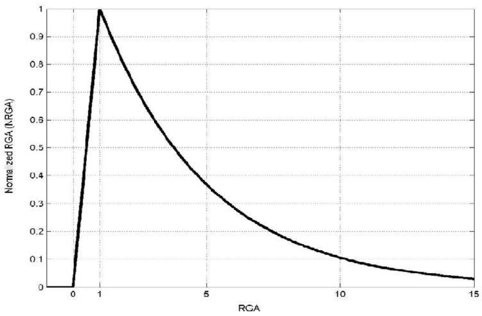

Fig.1 Normalization function for NRGA [10]

In this method the nonlinear mapping "close to 1" can be interpreted from the function shown in Fig.1The above function was applied to the elements of RGA matrix to obtain a new matrix which is called as NRGA with elements

$$\phi_{ij} = f(\lambda_{ij}) \quad (15)$$

The RGA pairing rules were interpreted by using NRGA in the following manner

Rule 1: Try to select pairs with large ϕ_{ij}

Rule 2: If the plant should be decentralized integral controllable (DIC) avoid selecting pairs with zero values ϕ_{ij}

Rule 3: For DIC the selected pairs should satisfy Niederlinski condition

$$NI = \frac{\det(G(0))}{\prod_{ps} g_{ij}(0)} \quad (16)$$

Where ps is the set of selected pairs

Using NRGA, the overall pairing measure was defined as-

$$\psi = max \sum_{ps} \phi_{ij} \quad (17)$$

Where ps is a complete pairing which satisfies NRGA Rules (1) and (2)

2.4. Relative Normalized Gain Array [RNGA]

A new loop pairing criteria based on the RNGA was proposed for control structure configuration [11]. The normalized gain [NG] for a particular transfer function was defined as-

$$k_{N_{ij}} = \frac{g_{ij}(j0)}{\tau_{arij}} \quad (18)$$

Where $g_{ij}(j0)$ is the steady state gain and τ_{arij} is a accumulation of the difference between the expected and real output of the normalized transfer function $g_{ij}(s)$.

Eq. 18 was extended to all the elements of the transfer function matrix $G(s)$ and the normalized gain matrix was obtained as-

$$K_N = G(j0) \odot T_{ar} \quad (19)$$

Where $\odot$ indicates element by element division. $T_{ar} = [\tau_{arij}]_{n\times n}$

The relative normalized gain [RNG] between output variable y_i and input variable u_j was defined as-

$$\phi_{ij} = \frac{K_{Nij}}{\hat{K}_{Nij}} \quad (20)$$

Here $\hat{K}_{Nij}$ is the effective gain between the output variable y_i and input variable u_j when all other loops are closed. The relative normalized gain array [RNGA] was calculated as-

$$\Phi = [\phi_{ij}]_{n\times n} = K_N \otimes K_N^{-T} \quad (21)$$

Some of the important properties of RNGA are-

1. The value of ϕ_{ij} is the measure of effective interaction expected in the i^{th} loop it its output y_i is paired with u_j.
2. The elements of RNGA across any row, or down any column, sum up to 1.

With this RGA-NI-RNGA based control configuration rules were developed as-

Manipulated and controlled variables in a decentralized control system in the following way that-

1. All paired RGA elements are positive
2. NI is positive.
3. The paired RNGA elements are close to 1.
4. Large RNGA elements should be avoided.

3. INPUT-OUTPUT PAIRING IN DECENTRALIZED CONTROL

A criterion for selecting input output pairs such that the resulting control structure is decentralized integral controllable [DIC] was proposed [12]. Several necessary conditions for DIC were presented in terms of steady state gain matrix. All the conditions presented were in terms of avoiding pairings where the plant gains may change signs as other loops are changed. Following rules were presented for pairing selection

Rule 1: eliminate pairings with negative RGA's

Rule 2: Eliminate pairings with negative Niederlinski Index.

Rule 3: Eliminate pairing with negative Morari indices of Integral Controllability which is given as-

$$MIC = Re\{\lambda(G^+(0))\} \tag{22}$$

Rule 4: Eliminate pairing with $Re\{\lambda(E(0))\} < 1; E = (G - G_{diag})G_{dia}^{-1}$

Rule 5: Eliminate pairings for which there exists a K (diagonal matrix with positive enteries) which yields $Re\{\lambda(G^+(0)K)\} < 0$

Analysis methods to determine achievable closed-loop system characteristics as a function of control system structure independent of controller design have been developed [13]. With these methods pairings which do not admit acceptable closed loop performance can be discarded before any controllers are designed. These methods only require the steady state knowledge of the plant. The concept of integral stabilizability and integral controllability was developed to study single loop controllers for multivariable plants.

Automatic decentralized control structure selection has also been studied [14]. The control structure selection problem is formulated as a special MILP employing cost coefficients. A disturbance free system was partitioned according to

$$\begin{bmatrix} y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ U_2 \end{bmatrix} \tag{23}$$

In this approach pairing y_1 and u_1 is considered and all other outputs Y_2 are assumed to be controlled by all other inputs U_2. When all other loops are open, i.e. $U_2 = 0$, the effect of u_1 on y_1 is given by

$$y_1 = g_{11}u_1 \tag{24}$$

When all other loops are closed and perfectly controlled

$$Y_2 = g_{21}u_1 + G_{22}U_2 \tag{25}$$

The value of the actuator U_2 becomes

$$U_2 = -G_{22}^{-1}g_{21}u_1 \tag{26}$$

In case of perfect control of all other loops the effect of u_1 on y_1 is given by

$$y_1 = (g_{11} + \hat{a}_{11})u_1 \tag{27}$$

The term $\hat{a}_{11} = -g_{12}G_{22}g_{21}$ is the indirect effect due to perfect control of all other loops.

The gain from u_1 to y_1 of the open-loop situation relative to the perfect control situation thus is-

$$\lambda_{11} = \frac{g_{11}(s)}{g_{11}(s) + \hat{a}_{11}(s)} = g_{11}(s)[G^{-1}(s)]_{11} \tag{28}$$

The relative interaction is given as-

$$\Phi_{11}(s) = \frac{\hat{a}_{11}(s)}{g_{11}(s)} = \frac{1}{\lambda_{11}(s)} - 1 \tag{29}$$

In general the relative interaction for the pairing of output y_i with input u_j is given as-

$$\Phi_{ij}(s) = \frac{\hat{a}_{ij}(s)}{g_{ij}(s)} = \frac{1}{\lambda_{ij}(s)} - 1 \tag{30}$$

Where-

$$\lambda_{ij} = \frac{g_{ij}(s)}{g_{ij}(s) + \hat{a}_{ij}(s)} = g_{ij}(s)[G^{-1}(s)]_{ij}$$

An interaction measure called as participation matrix was proposed which was based on the dynamic model of the process [15]. This index was built on the system gramians and also provides a measure of achievable performance of a given controller architecture with respect to either the full MIMO case or another controller architecture. The participation matrix is defined as-

$$\varphi_{ij} = \frac{trace[P_jQ_i]}{trace[PQ]} \tag{31}$$

Where-

P and Q are controllability and observability gramians.

A dynamic loop pairing criterion for decentralized control of multivariable processes was proposed by utilizing both steady state gain and band width information of the process open loop transfer function elements [16]. The loop pairing procedure of RGA is extended in this method by defining an effective gain matrix which can reflect dynamic loop interactions under finite bandwidth control. The effective gain matrix was obtained as-

$$E = G(0) \otimes \Omega \tag{32}$$

Here-

$G(0)$ is the steady state gain matrix and is the bandwidth matrix. The elements of matrix E, e_{ij} represents interaction energy to other loops when loop $y_i - u_j$ is closed. Bigger the value of e_{ij}, more dominant the loop will be. The effective relative gain between output variable y_i and the input variable u_j is then defined as-

$$\Phi_{ij} = \frac{e_{ij}}{\hat{e}_{ij}} \tag{33}$$

Where- $\widehat{e}_{ij}$ is the effective gain between output variable y_i and the input variable u_j when all other loops are closed. When the all effective relative gains are calculated for all the input/output combinations of a multivariable process, it results in an array of the form similar to that of RGA which is called as effective RGA [ERGA] which can be calculated as-

$$\Phi = E \otimes E^{-T} \quad (34)$$

In presence of plant uncertainties the input-output pairs in decentralized control structure can change. Input-output pairing in presence of plant uncertainties has been discussed [17]. Hankel interaction index array is proposed to choose appropriate input-output pairing in presence of plant uncertainties. Hankel interaction index array is calculated as-

$$\left[\bar{\Sigma}_H\right]_{ij} = \left[\bar{\sigma}\left(W_C^{ij} W_O^{ij}\right)\right] \quad (35)$$

Where W_C^{ij} and W_O^{ij} represents controllability and observability Gramians for the i^{th} input and j^{th} output, respectively. If there is no overlap between variation bounds of the same row and the same column in Hankel Index interactions array, the nominal input-output pairing remains valid for all parameter variations

Most of the methods used for selecting input-output pairs require evaluation of every alternative in order to find the optimal pairings. As the number of alternatives grows rapidly with problem size, pairing selection through exhaustive search becomes cumbersome. To overcome this difficulty a novel branch and bound [BAB] approaches for pairing selection using relative gain array and μ -interaction measure as a selection criteria was presented [18]. The pairing selection is formulated as the following optimization problem

$$\min_{P_n \in P(N_n)} J(P_n) \qquad s.t \quad L_i(P_n) \geq 0; \quad i = 1,2,...,l \quad (36)$$

Where-

P is the pairing selection criterion and $L_i(i = 1,2,....n)$ represents a set of inequality constraints

4. INPUT-OUTPUT PAIRING BASED ON INDICES

4.1 Niederlinski Index

Niederlinski is a fairly useful stability analysis method [2]. It can also be used to eliminate unworkable pairings of variables in the early stage of design. This method is used when integral action is used in all the loops and it uses only the steady state gains of the process transfer function matrix. It is a "necessary but not sufficient condition" for stability of a closed system with integral action. If the index is negative, the system will be unstable for any controller setting which is called as integral instability. It the index is positive, the system may be or may not be stable. Niederlinski index is defined as-

$$NI = \frac{Det[K_P]}{\prod_{j=1}^{N} K_{Pjj}} \quad (37)$$

Where-

K_P is the matrix of steady state gains from the process open loop transfer function.

K_{Pjj} are diagonal elements is steady state gain matrix.

4.2 Hankel Interaction Index

An interaction measure called as Hankel interaction index was proposed for stable multivariable systems [19]. This index is based on Hankel norm of the SISO elementary subsystems built from the original MIMO system. For each elementary subsystem Hankel norm is used to quantify the ability of input u_j to control output y_i. These norms are collected into matrix whose ij^{th} elements is given as-

$$\left|\Sigma_H\right|_{ij} = \left|G_{ij}(z)\right|_H \quad (38)$$

The normalized Hankel index array is given as-

$$\left|\Sigma_H\right|_{ij} = \frac{\left|G_{ij}(z)\right|_H}{\sum_{ij}\left|G_{ij}(z)\right|_H} \quad (39)$$

4.3 Passivity Index

An experimental pairing method for multivariable system was proposed which is based on passivity of the paired system [20]. A frequency dependent passivity index was introduced to characterize the total destabilizing effect of both loop interactions and process dynamics.

4.4 Zeta Ratio

The ratio the product of the diagonal elements to that of the diagonal elements of the steady state gain matrix is called as zeta ratio. Two input two output systems are characterized by this zeta ratio. The concept of zeta ratio was extended to higher order systems [21]. The steps involved in this approach are-

1. Generate all possible single loop control configurations
2. Evaluate the Niederlinski index of each control configuration
3. For the configurations with $NI > 0$, determine the ratio of the product of the

diagonal elements to that of the diagonal elements of the steady state gain matrix [ξ].

4. Sort the viable control configurations in order of increasing value of ξ. The one with the least value is referred to as the zeta ratio and gives the suitable configuration.
5. Evaluate the RGA matrix for this configuration.

5. INPUT OUTPUT PAIRING USING FUZZY LOGIC

In many industrial real problems, accurate system models are not easy to derive, such as complex manufacturing processes and chemical processes. Large number of variables in these processes often led to nonlinear interactions which are not easy to quantify with crisp numeric precision. The multivariable interaction analysis is difficult to execute of these systems without mathematical models. In recent years, fuzzy control technologies have attracted intense interest. During the past two decades different types of fuzzy controller design have emerged for the manipulation of multivariable systems with nonlinear characteristics, complex structure and uncertainties. Compared with traditional control techniques based on exact mathematical models, fuzzy-model-based controls are powerful and robust tools for control of ill-defined and complex systems.

The interaction analysis for multivariable systems based on system fuzzy model has been proposed [22]. The nonlinear multivariable system is modeled as fuzzy basis functions networks [FBFN]. Then a steady state gain array is calculated based on the FBFN around the specific operating point. The FBFN model developed is shown in Fig.2

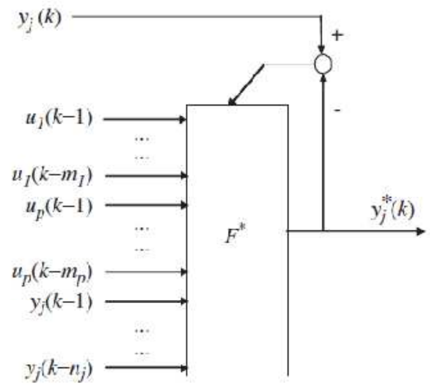

Fig. 2 Non-linear Dynamic System Modeling by FBFN [22]

$u_1(k-1).....u_1(k-m_1)......u_p(k-1).....u_p(k-m_p).y_j(k-1).....y_j(k-n_j)$ are the experimental input variables. $y_j(k)$ is the experimental output, $y_j^*(k)$ is the model output from FBFN and F^* is the resultant FBFN model.

The steady state input-output gains are given as-

$$g_{j1} = \frac{\sum_{t_1=1}^{m_1} \frac{\partial f_j}{\partial u_1(k-t_1)}}{1-\sum_{t_y=1}^{n_j} \frac{\partial f_j}{\partial y_j(k-t_y)}}$$

$$.g_{jp} = \frac{\sum_{t_p=1}^{m_p} \frac{\partial f_j}{\partial u_p(k-t_p)}}{1-\sum_{t_y=1}^{n_j} \frac{\partial f_j}{\partial y_j(k-t_y)}} \quad (40)$$

The system steady state gain array is obtained as-

$$G(0) = [g_{jr}] = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1p} \\ \vdots & \vdots & \vdots & \vdots \\ g_{q1} & g_{q2} & \cdots & g_{qp} \end{bmatrix} \quad (41)$$

The RGA is then calculated as-

$$\Lambda = G(0) \times [G(0)^{-1}]^T \quad (42)$$

A more accurate loop pairing method which utilizes both steady state and dynamic information of the system was proposed for MIMO system which were represented by Takagi-Sugeno (T-S) fuzzy models [23]. Each individual loop in the MIMO process is represented by a T-S fuzzy model based on the data and the models are then assembled to form the MIMO model. For an open loop stable and nonsingular at steady state MIMO system of n inputs and n outputs the following T-S fuzzy model was obtained

$$F_{TS} = [f_{TSij}]_{n\times n} = \begin{bmatrix} f_{TS11} & f_{TS12} & \cdots & f_{TS1n} \\ f_{TS21} & f_{TS22} & \cdots & f_{TS2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{TSn1} & f_{TSn2} & \cdots & f_{TSnn} \end{bmatrix} \quad (43)$$

From the definition of classical RGA, the relative gain for the above MIMO process was defined as-

$$\lambda_{TSij} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_{\Delta u_{r\neq j}=0}}{\left(\frac{\partial y_i}{\partial u_j}\right)_{\Delta y_{r\neq j}=0}} \quad (44)$$

Where λ_{TSij} is the relative gain for the loop $y_i - u_j$. $\left(\frac{\partial y_i}{\partial u_j}\right)_{\Delta u_{r \neq j}=0}$ is the open loop steady state gain for f_{TSij} and $\left(\frac{\partial y_i}{\partial u_j}\right)_{\Delta y_{r \neq j}=0}$ is the apparent process gain for f_{TSij} when all other loops are closed. Due to nonlinear nature of T-S fuzzy model the relative gain is calculated at the following operating point.

$$x_{0ij} = \left[u_{0j}(t-\tau_{ij}) \ldots u_{0j}(t-\tau_{ij}-p) y_{0i}(t-1) \ldots y_{0i}(t-q)\right]$$

Where p and q are sampling parameters and $\tau_{ij} = \frac{d_{ij}}{T}$, where d_{ij} denotes the time delay in the loop $y_i - u_j$ and T is the sampling interval and x_{0ij} is the input vector.

The steady state gain matrix for F_{TS} becomes-

$$K_{TS} = \left[k_{TSij}\right]_{n\times n} = \begin{bmatrix} k_{TS11} & k_{TS12} & \cdots & k_{TS1n} \\ k_{TS21} & k_{TS22} & \cdots & k_{TS2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{TSn1} & k_{TSn2} & \cdots & k_{TSnn} \end{bmatrix} \quad (45)$$

k_{TSij} is the steady state gain for f_{TSij} based on x_{0ij}

The relative gain for f_{TSij} is then defined

$$\lambda_{TSij} = \frac{k_{TSij}}{\hat{k}_{TSij}} \quad (46)$$

Where-

k_{TSij} is steady state gain for the loop f_{TSij}

$\hat{k}_{TSij}$ is steady state gain for the same loop when all other loops are closed.

The RGA for the T-S fuzzy model is then given as-

$$\Lambda_{TS} = K_{TS} \otimes K_{TS}^{-T} \quad (47)$$

The pairing rules are similar to the classical RGA rules.

The Niederlinski index (NI) for the T-S fuzzy model is defined as-

$$N_{TS} = \frac{\det[K_{TS}]}{\prod_{i=1}^{n} k_{TSii}} \quad (48)$$

$N_{TS} > 0$ is the additional pairing rule

The normalized integrated error is calculated as-

$$e_{TSij} = \sum_{r=0}^{\infty} \frac{y_i(\infty) - y_i(r.T)}{k_{TSij}} T \quad (49)$$

Where-

$y_i(\infty)$ is the steady state output

$y_i(r.T)$ is the output at the time of r^{th} sample of f_{TSij}

To combine the steady state gain and the normalized integrated error for the interaction measure and loop pairing, the normalized gain for f_{TSij} is defined as-

$$K_{NTSij} = \frac{k_{TSij}}{e_{TSij}} \quad (50)$$

The relative normalized gain is defined as-

$$\Phi_{TSij} = \frac{k_{NTSij}}{\hat{k}_{NTSij}} \quad (51)$$

Where-

k_{NTSij} is normalized gain for the loop f_{TSij}

$\hat{k}_{NTSij}$ is the normalized gain for the same loop when all other loops are closed.

The relative normalized gain array [RNGA] is then given as-

$$\Phi_{TS} = K_{NTS} \otimes K_{NTS}^{-T} \quad (52)$$

The RNGA-based control configuration rules are then proposed as-

1. All paired RGA elements are positive
2. NI is negative
3. The paired RNGA elements are positive
4. The paired RNGA elements are closet to 1
5. Large RNGA elements should be avoided

7. CONCLUSION

Thus it has been seen that complicated chemical processes are multivariable in nature where there are multiple inputs and multiple outputs. Designing control system for these processes is a complicated task as there are multiple control loops. The interactions among these control loops needs to be analyzed so as to have a satisfactory control system. The purpose of doing this analysis is to decide the input output pairing which is will minimize the interaction among various loops. This paper was devoted to review methods that have being used to accomplish this important task. The methods include various arrays proposed to decide the input output pairing which are based on the classical relative gain array. Many indices were also proposed to carry out the task of input output pairing. Some of the important indices were briefly discussed. Many methods were developed to find out input output pairing in a decentralized control structure which is most favored for multivariable systems. Some of these methods were also reviewed. Recently for complicated processes techniques like fuzzy logic have been used to model the processes. Many methods based on fuzzy logic are alos proposed in literature. Some these methods are also reviewed.

REFERENCES

[1] Bristol E. H. (1966). On a new measure of interaction for multivariable process control. *IEEE Trans. on Automatic Control, 11, 133-134*

[2] A. Niederlinski (1971). A heuristic approach to the design of linear multivariable interacting subsystems. *Automatica*, 7, 691-701

[3] McAvoy, T. J. (1983). *Interaction Analysis-Principles and Applications* [Instrument Society of America, Research Triangle Park, NC]

[4] Grosdidier, P. and M. Morari (1986). Interaction measures for systems under decentralized control. *Automatica*, 22(3), 309-319

[5] Aldo Balestrino and Alberto Landi (2006). On input output selection for multiloop control: From RGA to ROMA. ADCHEM, International Symposium on Advanced Control of Chemical Processes, Gramado, Brazil-April 2-5,2006

[6] Karlsmose, J., A. Koggershol, N. Jensen and S. B. Jorgensen (1994). A two stage procedure for control structure analysis and design. *Computers Chem. Eng.*, 18, S465-S470.

[7] Witcher, M., & McAvoy, T. J. (1977). Interacting control systems: Steady state and dynamic measurements of interactions. *ISA Transactions*, 16, 35-41

[8] Tung, L., & Edgar, T. (1981). Analysis of control-output interactions in dynamic systems. *A.I.Ch.E Journal*, 27, 690-693

[9] Thomas Mc Avoy, Yaman Arkun, Rong Chen, Derek Robinson, & P. David Schnelle (2003). A new approach to defining a dynamic relative gain. *Control Engineering Practice*, 11, 907-914

[10] A., Fatehi and A., Shariati (2007). Automatic pairing of MIMO plants using normalized RGA. *Proceedings of the 15th Mediterranean Conference on Control & Automation*, July 27-29, 2007, Athens Greece

[11] Mao-Jun He, Wen-Jian Cai, Wei Ni, Li-Hua Xie (2009). RNGA based control system configuration for multivariable processes. *Journal of Process Control*, 19, 1036-1042

[12] Sigurd Skogestad and Manfred Morari (1992). Variable selection for decentralized control. *Modeling, Identification and Control*, 13 (2), 113-125

[13] Peter J. Campo and Manfred Morari (1994). Achievable closed-loop properties of systems under decentralized control: Conditions involving the steady-state gain. *IEEE Transactions on Automatic Control*, 39 (5).

[14] John Bagterp Jorgensen and Sten Bay Jorgensen (2000). Towards automatic decentralized control structure selection. *Computers and Chemical Engineering*, 24, 841-846

[15] Mario E. Salgado and Arthur Conley (2004). MIMO interaction measure and controller selection. *Int. J. Control*, 77 (4), 367-383

[16] Qiang Xiong, Wen-Jian Cai and Mao-Jun He (2005). A practical loop pairing criterion for multivariable processes. *Journal of Process Control*, 15, 741-747

[17] Bijan Moaveni and Ali Khaki Sedigh (2008). Input-output pairing analysis for uncertain multivariable processes. *Journal of Process Control*, 18, 527-532

[18] Vinay Kariwala and Yi Cao (2008). Efficient branch and bound methods for pairing selection. *Proceedings of the 17th World Congress The International Federation of Automatic Control*, Seoul, Korea, July 6-11, 2008

[19] Bjorn Wittenmark and Mario E. Salgado (2002). Hankel-norm based interaction measure for input-output pairing, *15th Triennial World Congress*, Barcelona, Spain.

[20] Jie Bao, Kwong H. Chan, Wen Z. Zhang, Peter L. Lee (2007). An experimental pairing method for multi-loop control based on passivity. *Journal of Process Control*, 17, 787-798

[21] Ajayi T O. and I. S Ogboh, Determination of control pairing for higher order multivariable systems by use of multi-ratios. *International Journal of Scientific and Engineering Research*, 3 (3)

[22] Chengying Xu and Yung C. Shin (2007). Interaction analysis for MIMO nonlinear systems based on a fuzzy basis function network model. *Fuzzy Sets and Systems*, 158, 2013-2025.

[23] Qian-Fang Liao, Wen-Jian Cai, Shao-Yuan Li and You-Yi Wang (2012). Interaction analysis and loop pairing for MIMO processes described by T-S fuzzy models. *Fuzzy Sets and Systems*, 207, 64-76